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C.W.R. REPORT NO. 300-37

DESIGN PROCEDURE

FOR

TRANSPIRATION AIR COOLED

TURBINE BLADES

Supplement to Final Report   Contract NOas 56-495-c

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DEPARTMENT OF THE NAVY,
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Contract NOas 56-495-c

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CURTISS-WRIGHT CORPORATION
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C.W.R. REPORT NO. 300-37

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DESIGN PROCEDURE FOR TRANSPIRATION COOLED
GAS-TURBINE BLADES

Summary

Contained herein is the calculation procedure for determining the coolant flow required to maintain a given wall temperature in transpiration-cooled turbine blades. The basic method is not new; it has evolved from numerous modifications based upon the experience acquired during testing and evaluation of transpiration cooled configurations. The theory supporting the equations is included.

The calculation procedure presented in this report has been tried and has proven to be a useful tool. It is believed that continued testing will further the understanding of the influence of turbulent flow on the aerodynamic and heat transfer characteristics of this type of cooling. With refinement of the theoretical approach, improvement in the design procedure can be realized.

In order that potentially feasible configurations can be evaluated quickly and, therefore, economically, the calculation method has been programmed for the IBM 704 digital computor. A brief description of the methods of preparing input data for the machine and of the operation sequence employed by the machine is also included.

Object

1. To outline in detail the procedure used in the design of transpiration cooled turbine blades.

2. To evaluate the results of cascade testing for the purpose of confirming the theory supporting the equations in the calculation procedure and to incorporate those modifications deemed necessary for improving the accuracy of the calculation method.

3. To program the refined design procedure into the I.B.M. 704 Computer.

Conclusions

1. A refined design procedure has been worked out in detail to aid in the design of transpiration cooled turbine blades.

2. The refined design procedure has been programmed on the I.B.M. 704 Computer, thus enabling the evaluation of many cooling configurations heretofore considered impractical because of time and cost limitations.

Recommendations

1. Employ the procedures outlined herein in the design of transpiration air cooled turbine blades.

2. Continue cascade testing to further the correlation between theory and experimental data.

3. Utilize the I.B.M. 704 Computer program developed herein to facilitate the design of transpiration cooled configurations.

SYMBOLS

The following symbols are used in this report:

- A Cross-sectional area, sq. ft.
- a Coefficient in linear spanwise variation of cooling-air temperature, $T_c = T_{c,r}$ ax
- b Chordwise peripheral width of coolant passage, ft
- b' Skin area associated with a particular passage and cross section, sq/ft
- β Angle between blade axis and rotor radius sector, degrees
- C_p Specific heat at constant pressure, $\frac{Btu}{Lb\,^{\circ}R}$
- D Leading edge diameter, ft; D_h Hydraulic diameter of coolant passage, ft
- d Derivitive
- Eu Euler number of gas flow over blade, $\frac{y}{w}\frac{dw}{dy}$
- F Friction force, lbs
- f Coolant passage friction factor; f_w Coolant flow parameter for laminar flow region
- g Acceleration due to gravity, ft/sec^2
- h Heat-transfer coefficient, $\frac{Btu}{hr\ ft^2\ ^{\circ}R}$
- K Permeability coefficient, sq. ft.
- k Thermal conductivity, $\frac{Btu}{ft\ ^{\circ}F\ sec}$
- L Length from stagnation point to trailing edge along either surface, ft
- m Mass coolant flow, Lb/sec
- Nu Nusselt number, hl/k
- ω Angular velocity, sec^{-1}; $\Omega = \frac{A\varepsilon C_p}{h}$
- p Pressure
- Pr Prandtl number, $\frac{c_p\mu}{k}$
- ρ Density, Lb/ft^3
- q_g Heat transfer by convection to blade skin from hot gas

q_r Heat transfer by radiation

q_s Heat transfer by sidewise convection in the coolant boundary layer

q_w Heat transfer by conduction through blade wall

Re Reynolds number defined by $\frac{\rho W y}{\mu_w}$

Re_0 Reynolds number defined by $\frac{\rho U D}{\mu_w}$

r Radius, ft; $r' = \frac{8.11}{(Re_0)^{.1}\left(\frac{y}{L}\right)^{.1}\left(\frac{\rho_w U_s}{\rho_0 U_{s,0}}\right)^{.1}}$

R Gas constant, ft - Lb/Lb °F

T Temperature °R

τ Porous blade - shell thickness, ft

T_{ad} Adiabatic gas temperature, °R

U Free stream gas approach velocity upstream of blade leading edge, ft/sec

U_s^* Dimensionless mass velocity in free stream, $\frac{\rho_w U_s}{\rho_0 U_{s,0}}$

v Cooling air velocity through porous wall, ft/sec

μ Absolute viscosity (based on porous wall temperature), Lb/ft. sec

W Gas velocity relative to blade, ft/sec

W_i Coolant velocity in passage, ft/sec

x Spanwise distance in coolant passage from blade root, ft

y Peripheral distance from stagnation point along either surface of blade, ft

y^* Dimensionless distance from stagnation point along blade, y/L

z Distance normal to skin in the gas direction, ft

Subscripts:

c Cooling air or cooling air passage

c,i Denotes coolant into air passage

c,o Denotes coolant out of air passage

e External gas flow

i Internal coolant flow

o Refers to fixed point in stream (e.g., upstream condition)

r Coolant passage entrance (blade root)

s Stream

w Porous wall

C.W.R. REPORT NO. 300-37

Introduction

As analytical theory has predicted, testing has shown that transpiration cooling is a very effective method of cooling gas turbine blades. This report intends to provide a better understanding of the transpiration cooling process by summarizing the design procedure that has evolved during five years of usage.

Inasmuch as porous materials currently available for use in transpiration cooled turbine blades do not possess sufficient strength to withstand the high stresses imposed by rotative turbine speeds, an internal load carrying "strut" is required to which the porous skin is attached. In addition to serving as the blade supporting member, the strut divides the cross-section of the blade into compartments.

Because of the acceleration of the gasses through the rotor, there exists around the periphery of an individual blade a substantial variation of static pressure. The amount of coolant flow that passes through the porous wall is dependent on the difference in the squares of the absolute static pressure levels on opposite sides of the wall with the result that small changes in pressure level can cause very large changes in coolant flow rate. It has also been observed that the temperature distribution changes drastically as the coolant flow rate is varied. Thus, the difficulty in achieving uniform temperature around the periphery of a blade is readily seen.

The optimum design for a transpiration-cooled, strut-supported turbine rotor blade as regards coolant flow, therefore, incorporates a blade skin with a variable chordwise and a variable spanwise permeability. This is attributed mainly to the fact that the large variations in external gas pressure and temperature give rise to similarly large variations in coolant demand. At a specific chordwise position, for example, the midsection of the blade may "see" a gas temperature 200°F higher than the tip section, and to make matters worse, a higher static pressure level. Thus, for a given coolant pressure level, a material with constant permeability would have to be chosen to satisfy the maximum coolant requirement with consequent wastes of coolant in the areas of lesser demand. As the peripheral size of the passages decreases, these effects in the chordwise direction become less poignant except in the leading and trailing edge areas.

The problems incurred in the fabrication of blade skins with variable permeability are still found prohibitive. Consequently, the underlying principle in the procedure is to assume a permeability and coolant supply pressure level which will yield satisfactory wall temperatures.

The design method presented herein prescribes a uniform skin temperature equal to the maximum allowable for the material. In this way, the coolant demand can be held to a minimum. The division of the blade interior into separate passages and the use of sized orifices in the blade base to meter coolant to the various passages tend to minimize local overcooling and total bleed requirement.

This report attempts to set down an outline of the basic principles of transpiration air-cooled blade design. It deals only with the thermodynamic aspect of cooling methods. No consideration is given to the important problems associated with stress analysis and fabrication. Moreover, the effects of changes in engine speed, flight speed, and altitude on pressure levels and pressure distributions around the periphery of turbine blades is not discussed. However, it should be stated here that inasmuch as the permeability of transpiration cooled blades must be designed to provide the proper cooling at a prescribed pressure distribution, variations of flight speed, engine speed, and altitude may cause the permeability to be inconsistent with the aerodynamic pressure distribution. The uniform design temperature will no longer prevail and in the process of providing adequate coolant to the hottest portion of the blade, excess coolant flow will result in the cooler areas.

Results

As progress is made in predicting convection heat transfer coefficients, the design procedure outlined in this report can be steadily improved. The reliability of any such method, however, can be established only through comparison of the results of well instrumented tests with predicted values. In this way, modifications of the theory can be made so that a fairly good design method is assured.

The cascade data developed under Contract NOas 56-495-c (Reference 8) should be compared with the results predicted by the method presented herein. It is unlikely that agreement will be exact in light of the many assumptions made in the analytical approach and the errors inherent in the experimental work, but a study of this kind can aid in establishing the limitations of the present approach.

Since computations can be made rapidly on the 704 computor, some interesting design studies can be made which could hardly be justified if manual calculation were employed. For instance, the chordwise temperature variation along the skin of coolant passages in regions where the external gas pressure changes rapidly is of interest. The most important example is the leading edge passage. The calculations can be made by dividing the passage into a number of fictitious passages with identical coolant pressure distributions. Actually, this is the only way to determine the total coolant requirement for such a passage.

The analytical determination of the heat that must be removed from the blade skin at the stagnation point along the leading edge does not take into consideration the heat conduction within the skin to cooler areas (on the pressure and suction sides). This safety factor will have an equalizing effect on the chordwise distribution of the blade skin temperature.

A resume of the design procedure as described herein follows:

1. Assume a value for the internal static pressure at the root section ($P_{i,r}$) and calculate the static pressure at any spanwise position ($P_{i,x}$) from equation (5).

2. Assume a value for the blade wall temperature (T_w) and calculate inside gas velocity (v) from equation (9).

3. Calculate coolant flow parameter (f_w) from equation (19) using the calculated value of (v). (A modified equation for (f_w) is used at the leading edge (1)).

4. Calculate Euler number (Eu) and the ratio of adiabatic gas temperature to wall temperature (T_{ad}/T_w).

5. For the laminar flow region, obtain $\frac{T_w - T_c}{T_{ad} - T_c}$ from Figure (4), and calculate T_w. For the turbulent region use equation (24) to calculate T_w.

6. If the calculated value of T_w does not check the assumed value, the calculation is repeated for a new T_w. If the values obtained for T_w are now compatable with design requirements, a new $p_{i,r}$ should be tried.

7. When a satisfactory skin temperature has been obtained at all locations for which a calculation has been made, the mass flow (ρv Lb./sec. Ft^2) is determined at each location from equation (9).

8. The coolant requirement for each passage, m_c is obtained by summing the product of each ρv and associated skin area over the entire passage. The sum of the m_c quantities for all passages represents the total coolant requirement per blade.

9. The equations required to obtain the orifice area necessary for the specified $p_{i,r}$ are contained in reference (1).

Discussion

Statement of the Design Problem

For a specified set of turbine design parameters, i.e. rotor speed, gas temperature and pressure, blade geometry, coolant temperature and pressure, etc., the temperature of the transpiration cooled blade skin will depend upon the porosity distribution of the skin and the pressure distribution in the coolant passages. The problem can be formulated in three ways:

I. Determine the required coolant pressure distribution to give a uniform wall temperature with a specified porosity distribution.

II. Determine the porosity distribution required with a specified coolant pressure distribution to give a constant wall temperature.

III. Determine the wall temperature distribution resulting from specified pressure and porosity distributions.

These three cases are summarized in Table I.

Design Aspects

Inasmuch as the coolant pressure distribution cannot exceed the compressor outlet pressure, and must increase from root to tip in a rotor blade, it cannot be varied indiscriminately; hence, Case I is not a very practical way to proceed. Manufacturers of porous material have yet to provide a convincing demonstration of their ability to provide a well controlled variation in porosity. Accordingly, Case II is not yet practical. The best approach appears to be defined by Case III. The porosity may be specified constant and the temperature distribution determined for a particular pressure distribution which is possible to obtain. If the temperature so calculated is unsatisfactory, a new pressure distribution can be determined and the process repeated.

If a satisfactory skin temperature cannot be obtained with the maximum coolant pressure available, at some available porosity, consideration may be given to adding an auxiliary compressor to increase the coolant pressure and/or to including a ram-air cooler to reduce the coolant temperature.

Analysis

The pressure distribution in the blade passages is calculated after assuming a value for the coolant pressure at the blade root. The root pressure can be made less than the coolant supply pressure by the insertion of an orifice in the blade root.

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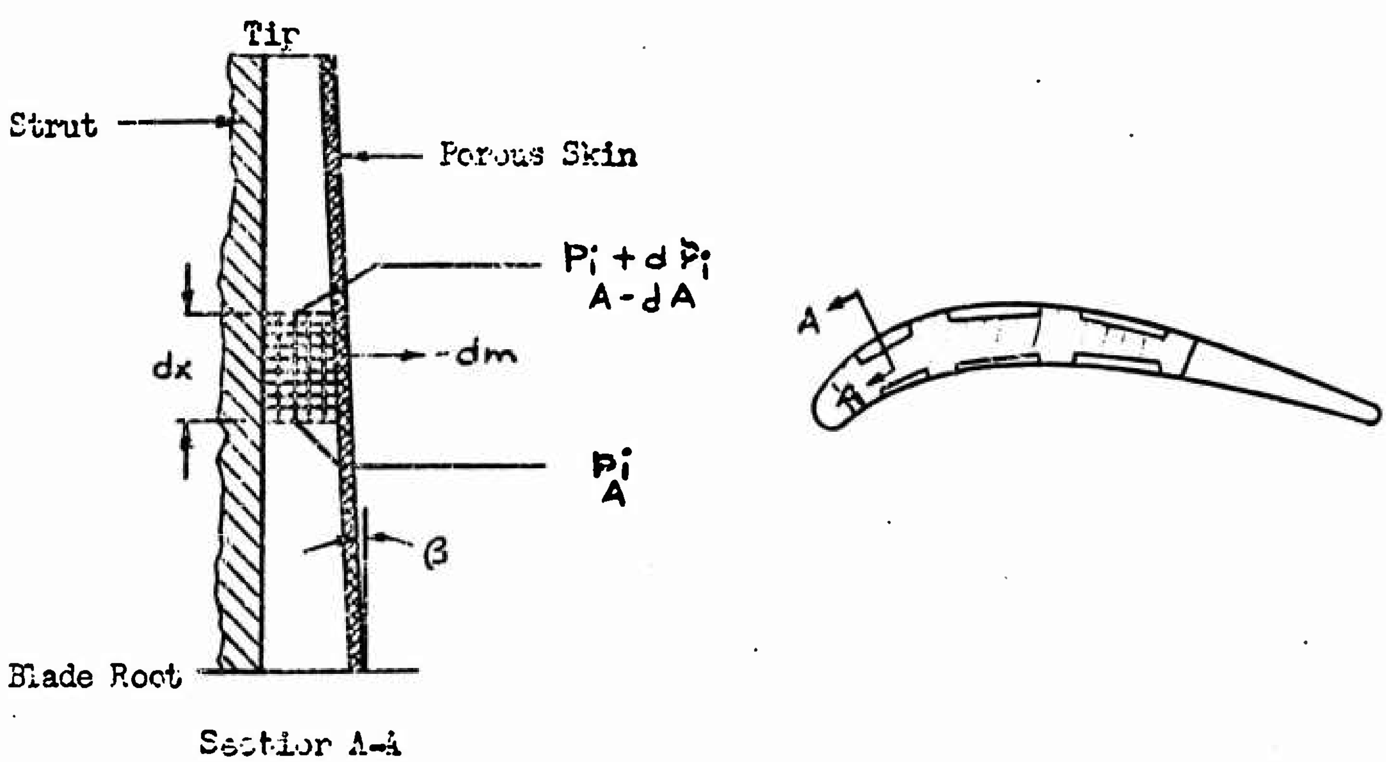
TABLE I

Case	Specified	Calculated
I	Wall temperature, T_w Permeability, K/γ	Coolant pressure Distribution, $P_i\ (x)$
II	$P_i\ (x)$ T_w	K/γ
III	$P_i\ (x)$ K/γ	T_w



ROTOR BLADE COOLANT PASSAGE

Figure 1

The pressure along the passages of stator blades will remain unchanged even at relatively high flow rates. This was verified by the cascade test of stator blades developed under NOas 56-495-c (Ref. 8). Rotor blades differ, however, since the centrifugal force must be considered. The calculation method according to Reference 3 is reviewed briefly here.

Consider a rotor blade channel of varying cross sectional area as illustrated in Figure 1.

Assume steady flow through the channel, and write Newton's second law for the fluid in the cross hatched volume element of the channel. This law must be applied to a control volume that is at rest relative to the channel.

Newton's second law states that the summation of forces acting on a body in a given direction must equal the rate of change in momentum of the body in this direction. In this case it is applied to the flow direction. The sum of all pressure forces is:

$$p_i A - (p_i + dp_i)(A - dA) - (p_i + \frac{dp_i}{2})\, dA$$

The third term comes about because the pressure at the wall inclined to the flow direction has a component in the flow direction which is approximately equivalent to the average pressure on the wall $(p_i + \frac{dp_i}{2})$ multiplied by the wall area and the sine of the angle β. The wall area multiplied by sine β is equal to dA. Neglecting second order terms, the total pressure force becomes $-A\,dp_i$.

The friction force can be expressed by the Fanning Equation as

$$F = \frac{-f\rho W_i^2 A\,dx}{2D_h}$$

The body force, caused by centrifugal acceleration of the fluid particles, can be written as

$$\rho A r \omega^2 \cos\beta\, dx$$

The change in momentum of the stream flowing through the control volume is

$$dm W_i = m\,dW_i + W_i\,dm$$

Equating the summation of the force terms to the change in momentum yields

$$-A\,dp_i + \rho A r \omega^2 \cos.\beta\, dx - \frac{\rho f W_i^2 A\,dx}{2d_h} = m\,dW_i + W_i\,dm \qquad (1)$$

The number of variables in Equation (1) can be reduced by use of the continuity equation

$$m = \rho A W_i \tag{2}$$

to eliminate W_i, and the equation of state

$$p_i = \rho R T \tag{3}$$

to eliminate ρ. Additional simplification results from introducing the relation

$$\frac{dm}{dx} = \frac{A\,d\left(\frac{m}{A}\right)}{dx} + \frac{m}{A}\frac{dA}{dx} \tag{4}$$

As a result of these substitutions, Equation (1) becomes

$$\left[1-\left(\frac{m}{A}\right)^2\frac{RT}{p_i^2}\right]\frac{dp_i}{dx} = \frac{p_i}{RT}r\omega^2\cos\beta - \frac{2RT}{p_i}\left(\frac{m}{A}\right)\frac{d\left(\frac{m}{A}\right)}{dx} - \left(\frac{m}{A}\right)^2\left(\frac{R}{p_i}\right)\left(\frac{dT}{dx}+\frac{T}{A}\frac{dA}{dx}+\frac{fT}{2D_h}\right) \tag{5}$$

In Equation (5), the specific mass flow m/A, the internal pressure p_i, the radius r, the cross sectional area A, and the temperature T are generally functions of the distance x from the channel entrance measured along the channel axis. The x dependence of A and r is easily determined. The coolant temperature is assumed to increase linearly according to

$$T = T_r + ax \tag{6}$$

Where "a" is an estimated quantity. Flow velocities are usually low enough in coolant passages so that the total temperature can be used instead of the static temperature.

A second expression including the same variables as functions of the distance from the passage entrance can be obtained from a consideration of the pressure drop through the porous wall when the pressure distribution along the outside of the passage is prescribed. The mass velocity of the coolant ejected through the porous wall at any location multiplied by the width of the porous portion of the channel wall measured in the circumferential direction equals the rate of change of the coolant mass flow passing through the passage at that location; that is,

$$b(\rho v) = -\frac{dm}{dx} \tag{7}$$

The quantity ρv is the average mass-flow rate over the channel width b. The negative sign appears in the right member of equation (7) since m decreases with increasing distance along the coolant passage. The mass velocity of a gas flowing through a porous skin is related to the pressure drop by

$$\frac{p_i^2 - p_e^2}{\tau} = C_1 (2RT\rho v)(\rho v) + C_2 \, 2RT (\rho v)^2 \qquad (8)$$

No general relationship for the parameters C_1 and C_2 has been obtained as yet so that they have to be determined by experiments for each porous material. Equation (8) can be simplified to

$$\rho v = C_K (p_i^2 - p_e^2)^n \qquad (9)$$

where

$$C_K = 3.050 \left(\frac{K}{\tau} \frac{1}{\mu T_w}\right)^n \qquad (10)$$

For a typical wire cloth, n = 5/8. Substituting equations (4) and (9) into equation 7 gives the desired second relation:

$$A \frac{d\left(\frac{m}{A}\right)}{dx} + \frac{m}{A} \frac{dA}{dx} = -bC_K (p_i^2 - p_e^2)^n \qquad (11)$$

Equation (5) and (11) must be solved simultaneously to yield $p_i(x)$. When the local coolant rate is prescribed, a direct solution is possible. However, the more important case in which the local coolant rate is a quantity to be determined requires a trial and error solution.

The iteration process can be avoided by an approximate approach which assumes the friction force and the rate of change of momentum are negligible. That such an assumption is justified can perhaps be better understood through consideration of an analogy to the present case.

If a vertical standpipe is filled with a liquid, a significant pressure increase is present from top to bottom because of the body force exerted by the dense liquid. If a standpipe were filled with gas, the pressure variation would be negligible. However, if the pipe were rotated at turbine speeds, the body force on the gas will increase to such an extent that a pressure gradient similar to that in a liquid exists.

Now suppose that holes are punched in the standpipe containing the liquid and that a supply of liquid is available at the top of the pipe sufficient to make up for liquid lost through the holes. If the flow is not large, the pressure distribution will not be much different than it was with the holes absent. As the flow rate is increased either by adding more holes and/or increasing the supply pressure, the velocity of the fluid in the pipe becomes large so that the friction and momentum terms in equation (1) must be included.

The simplified equation becomes a force balance,

$$-A\,dp_i + Ar\omega^2 dx = 0 \qquad (12)$$

which can be directly integrated to give:

$$\ln\left(\frac{p_i}{p_{i,r}}\right) = \frac{\omega^2 T_{c,r}}{g R a^2}\left(\frac{a r_r}{T_{c,r}} - 1\right)\ln\left(1 + \frac{a x}{T_{c,r}}\right) + \frac{\omega^2 x}{g R a} \qquad (13)$$

Calculations using both the approximate and the rigorous approach for rotor blade cooling passages show that large errors are not introduced by the approximate method even at large coolant flow rates.

The coolant mass flow rate required to give a specified local value of the skin temperature cannot be obtained directly from equation (9) since the skin temperature, T_w, which appears in this equation depends upon the relative rates at which heat is added by the gas and removed by the coolant. Therefore, another equation relating T_w and ρv must be obtained from a heat balance at the position in question.

A section of the porous skin is illustrated in Figure 2. Heat can enter the bounded region by convection from the hot gas, q_g; from the entering coolant, $q_{c,i}$; by conduction along the porous wall, q_w; from sidewise convection in the coolant boundary layer, q_s; and by radiation, q_r. Heat leaves the bounded section in the exit coolant, q_c. Heat transfer to turbine blades by radiation is usually negligible, and if the sidewise heat transfer terms are also neglected, a heat balance on the bounded section becomes:

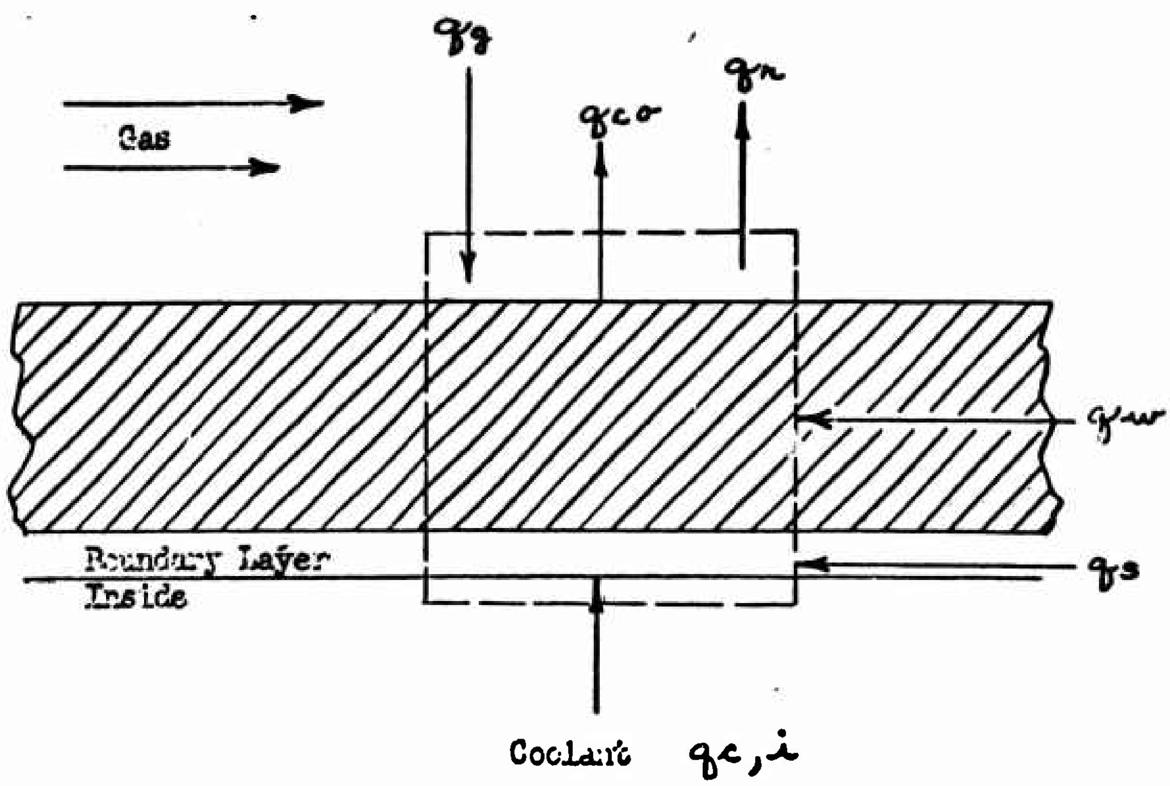
$$h\left(T_{ad} - T_w\right) dA = C_p \rho_w v \left(T_w - T_c\right) dA \qquad (14)$$

The first term represents the convection heat transfer from the gas and the term on the right the net heat picked up by the coolant. Equation (14) can be rewritten in the form:

$$\rho_w v = \frac{h\left(T_{ad} - T_w\right)}{C_p\left(T_w - T_c\right)} \qquad (15)$$

In order to use equation (15) in conjunction with equation (9) to determine the coolant flow and Tw, and expression for the heat transfer coefficient h is the most difficult problem of the entire analysis.

The heat transfer coefficient depends upon the character of the boundary layer and therefore will change markedly when the boundary layer changes from laminar to turbulent. The heat flow from the gas boundary layer on the outside of the blade into the blade surface is given by



CROSS SECTION THROUGH PART OF BLADE WALL

Figure 2

$$q_g = k\left(\frac{\partial T}{\partial z}\right)_w dA = h\left(T_{ad} - T_w\right) dA \tag{16}$$

Equation (16) can be solved for h to give:

$$h = \frac{k\left(\frac{\partial T}{\partial z}\right)_w}{\left(T_{ad} - T_w\right)} \tag{17}$$

which shows h to be a function of the temperature gradient at the wall and the difference between the gas and wall temperatures.

Heat Transfer Through Laminar Boundary Layer

For the laminar region, an exact solution of the boundary layer equation can be obtained to yield $\left(\frac{\partial T}{\partial z}\right)_w$. This exact approach is difficult and tedious. The uncertainties which still are connected with the calculation of heat transfer on turbine blades at present do not justify the time expenditure necessary for exact calculations and suggest approaches which derive approximate heat-transfer parameters with a small effort.

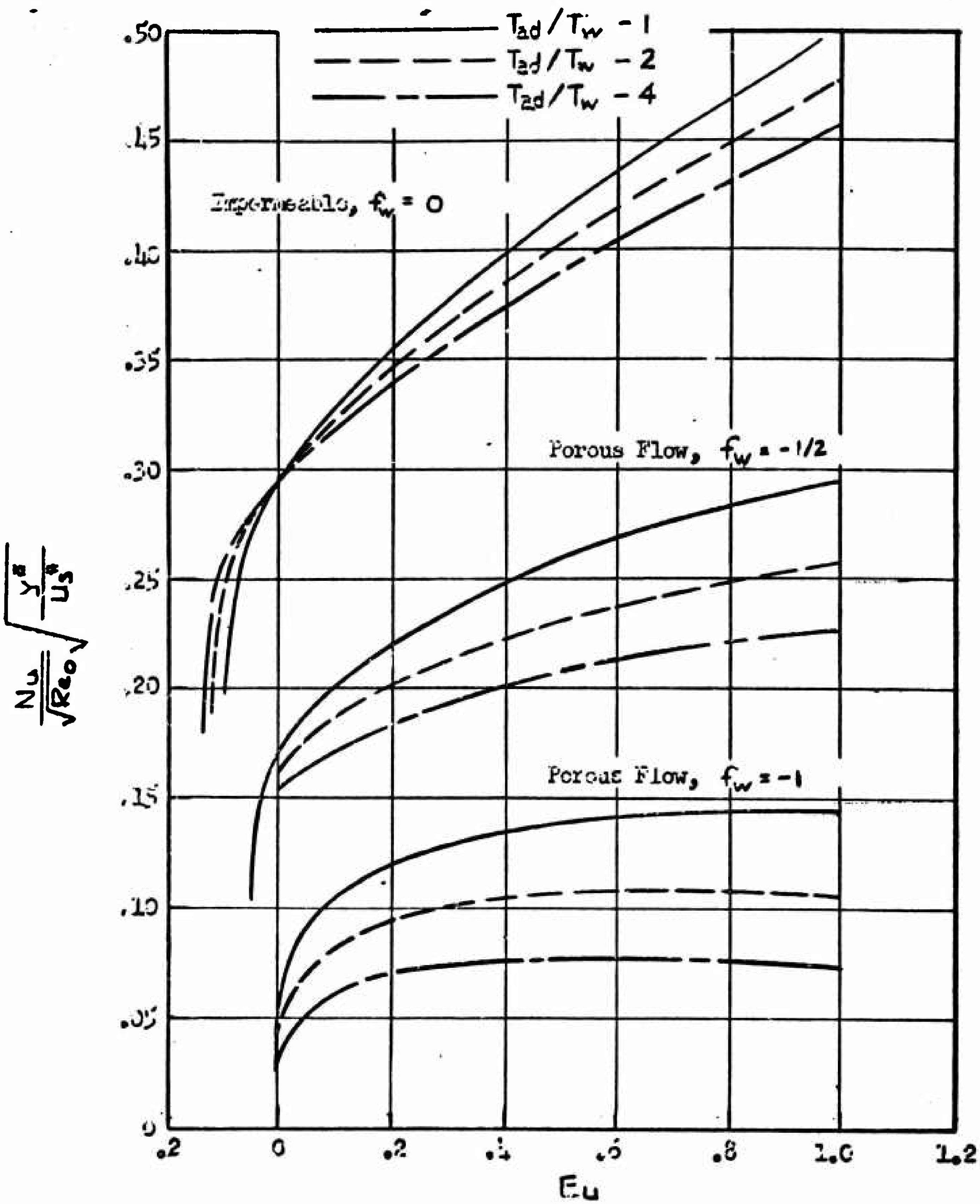
In Reference 5 the calculation of local heat-transfer coefficients from approximate solutions to the laminar boundary layer equations around a porous wedge was shown. From these results for porous wedges, local heat-transfer coefficients for the laminar region of a gas turbine blade can be obtained by the stipulation that the local heat-transfer coefficient at a specified point along the blade periphery is identical to that on a wedge for which, at the same distance from the stagnation point, the Euler number is the same as that on the blade. The Euler number is given by:

$$Eu = \frac{Y}{U}\frac{\partial W}{\partial Y} \tag{18}$$

The results of Reference 5 calculations are plotted in Figure 3 which presents a relation between the heat-transfer coefficient contained in Nu, the coolant velocity contained in f_w, and the known flow parameters. In this figure the external and coolant flows are assumed to have equal fluid properties and $Pr = 0.7$; calculated for small mach numbers. The coolant flow parameter, f_w, is given by:

$$-f_w = \frac{2}{Eu+1}\frac{V}{W}\sqrt{Re} \tag{19}$$

The magnitude of $-f_w$ is an indication of the rate of coolant flow.



LAMINAR HEAT TRANSFER PARAMETER
VERSUS
EULER NUMBER FOR IMPERMEABLE & PERMEABLE WEDGES.

Figure 3

In the discussion thus far, the following relationships have been established:

(a.) $p_i = f(x)$ From (5)

(b.) $\rho v = f(p_i, T_w)$ From (9)

(c.) $\rho v = f(h, T_w)$ From (15)

(d.) $h = f(v, T_w)$ From (Fig. 13)

This system of equations contains four unknowns pi, Tw, v, h; and can be solved for the desired quantities, Tw and v. Since the relation in (d) is presented in graphical form, the solution is a trial and error process.

In order to facilitate calculations, Figure 4 was prepared which eliminates h from equations (c) and (d). The calculation procedure requires evaluation of p_i from (a) and an iteration process between (b) and Figure 4.

Heat Transfer Through Turbulent Boundary Layers

No exact solutions exist for the case in which the boundary layer is turbulent. Experimental information is limited on the effect of pressure variations. It is known, however, that pressure variations have less influence in the turbulent flow region on solid surfaces than in the laminar flow region. For flow with constant pressure, an approximate method developed in Reference 6 and later simplified by Reference 7 checks experimental results fairly well. The assumptions made in Reference 7 are:

1. The flow consists of a turbulent region and a laminar sublayer which separates the turbulent flow from the wall surface.

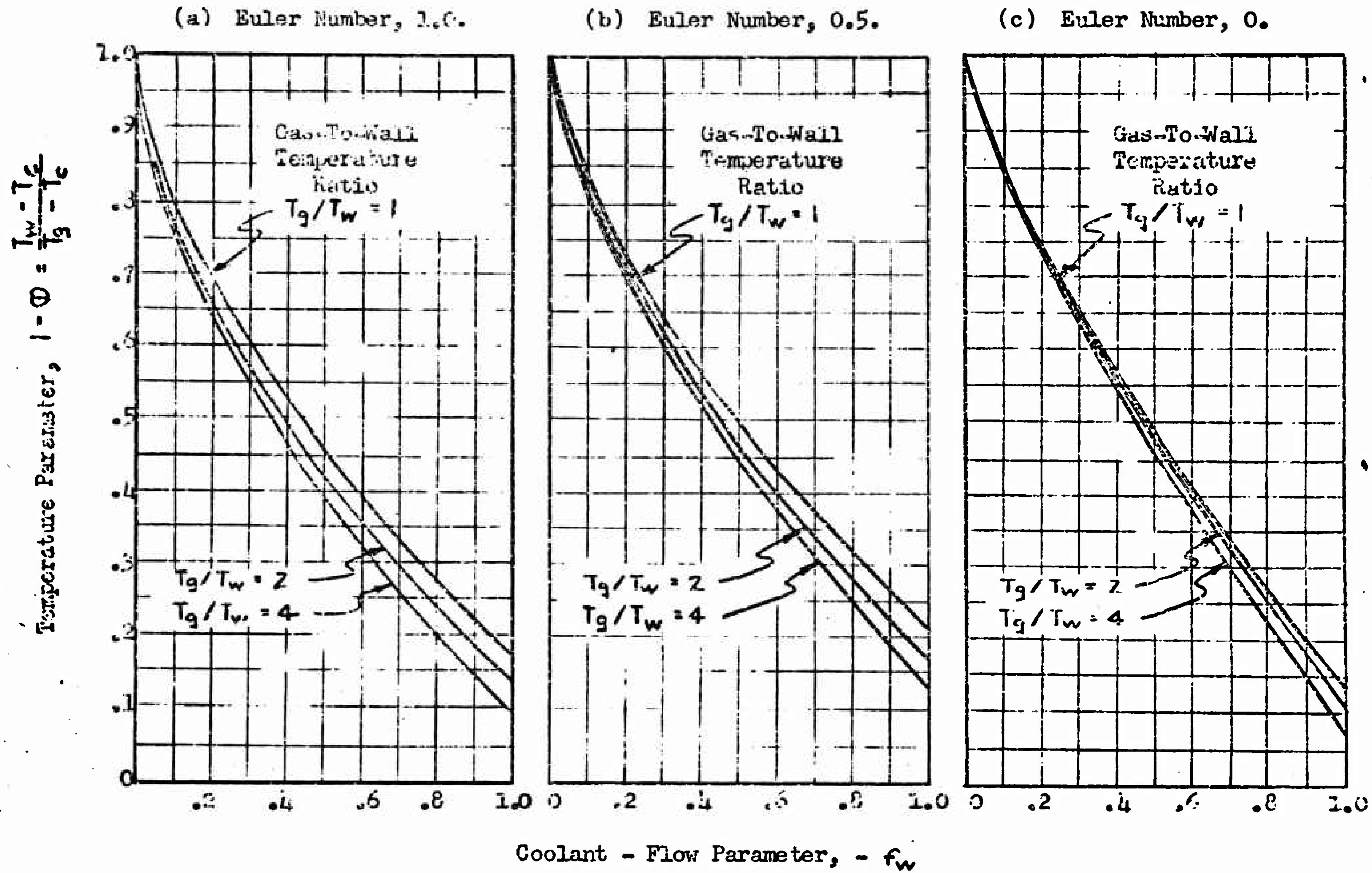
2. The temperature and velocities in the turbulent region have the same values on a transpiration cooled wall as in an ordinary boundary layer on a solid surface under otherwise identical conditions.

By restricting the investigation to fluids with a Prandtl number near unity, the following expression for the wall temperature is obtained in Reference 7:

$$\frac{T_w - T_c}{T_{ad} - T_c} = \frac{r'}{e^{r'\Omega} + r' - 1} \tag{20}$$

where r' is the ratio of the velocity parallel to the surface at the border between the laminar sublayer and the turbulent part of the boundary layer, to the stream velocity outside the boundary layer and where

$$\Omega = \frac{\rho_c v_c c_p}{h} \tag{21}$$



COOLANT FLOW AND TEMPERATURE PARAMETERS OF TRANSPIRATION-COOLED WALL IN LAMINAR GAS-FLOW REGION. PRANDTL NUMBER FOR AIR, 0.7.

Figure 4

with h the heat-transfer coefficient that would be present on a solid surface under the same outside flow conditions. The equation for turbulent heat transfer on a solid flat plate can be used in determining h.

Under these conditions

$$\Omega = \frac{v}{v_w} \frac{P_r^{2/3}}{.0296} (Re_0)^{1/5} \left(\frac{Y}{L}\right)^{1/5} \left(\frac{\rho_w U_s}{\rho_c U_{s,c}}\right)^{1/5} \tag{22}$$

Moreover, for a flat plate

$$r' = \frac{2.11}{(Re_0)^{.1} \left(\frac{Y}{L}\right)^{.1} \left(\frac{\rho_w U_s}{\rho_0 U_{s,c}}\right)^{.1}} \tag{23}$$

Substitution of equations (22) and (23) into equation (20) and simplification yields

$$\frac{T_w - T_c}{T_{ad} - T_c} = \frac{\dfrac{2.11}{Re^{0.1}}}{\exp\left[\dfrac{7.3 (\rho v) P_r^{2/3} Y}{Re^{0.9} \mu}\right] + \dfrac{2.11}{Re^{0.1}} - 1} \tag{24}$$

Equations (a) and (b) which were used to obtain a solution for the laminar flow region are also used for the region over which the boundary layer is turbulent. However, equations (c) and (d), for which Figure (4) was substituted in the laminar flow case, are now replaced by equation 24.

The Computor Program

Because of the tedious and time consuming iterative process involved in the calculations through equation (7), this portion of the method has been programmed for the 704 computor. In order to establish a systematic scheme for the machine calculations, the coolant requirements and skin temperatures for all of the passages in each blade section are evaluated before going to the next section. At each section the calculations are made for the leading edge, the laminar region, and the turbulent region respectively.

This problem was prepared for the IBM 704 digital computor using the FORTRAN system.

For a given set of inputs corresponding to a particular airfoil section, the computor program begins with the leading edge calculations. Utilization of a bivarient map is made which given $(1 - \Phi)$ as a function of f_w and T_0/T_w assuming Eu = 1.0. This map was tabulated so that the program could obtain values of $(1 - \Phi)$ by interpolation. The order of interpolation on f_w was 2 and on T_0/T_w was 1. A value for T_w was assumed and the equations located in the analysis were solved to obtain a secord value of T_w. The steps were then repeated using this T_w as the assumed T_w. The process was repeated until the calculated and assumed T_w's agreed to within 4 degrees. An upper limit of 30 iterations was fed into the program.

The temperature profile in the laminar region was then calculated. A bivarient map for $Eu = 1.0$ plus other similar maps for $Eu = .5$ and $Eu = 0$ were then used to obtain $(1 - \varphi)$. A value of $(1 - \varphi)$ was obtained from each of the maps in the same way as in the first section. Once Eu was known, second order interpolation could then be used on the three values to obtain $(1 - \varphi)$. T_w was iterated on just as in the first section except that convergence is not always obtained after 30 iterations. This occurs whenever f_w is greater than 1. As this condition is physically meaningless, T_w and m_c were printed as zero when convergence was not attained.

The third segment of the program deals with the turbulent flow region. It utilizes an analytic expression for $(1 - \varphi)$ rather than maps. T_w is iterated on as in the first two steps.

The computor time for any given set of input parameters is about one minute.

The sample data used to "debug" the computor program are contained in Table II.

The calculated results for these data are presented in Table III in the form that is printed out by the machine. The symbols printed by the computor and the corresponding notation as used in this report are both indicated in the column headings of Table III.

2 (No. of x's) 3 (No. of y's)

.017 (X_1) .155 (X_2)

.0085 (D_1) .0085 (D_2)

850.(ω) 1000.($T_{c,r}$) 1025.(a) 1.(r_r) .7 (P_r) 1400.(V) 1*

5.(m) -10.(n) (where $K/\tau = m \cdot 10^n$)

x	y	$P_{i,r}$	W	T_g	P_e	b'	$\frac{\partial W}{\partial y}$
.017	.0	11500.	0.	2300.	10850.	.00292	0.
.017	.0354	11000.	1080.	2400.	9500.	.00209	600.
.017	.0866	10500.	1900.	2300.	7520.	.00306	1780.
.155	.0	11500.	0.	2300.	10850.	.00292	0.
.155	.0354	11000.	1080.	2400.	9500.	.00209	600.
.155	.0866	10500.	1900.	2300.	7520.	.00306	1780.

* Calculation identification number

Typical input data supplied to the I.B.M. 704 Computer in the form used by the key puncher.

TABLE II

LAMINAR FLOW REGION   X = 0.01700   D = 0.00850

Y (y)	TW (T_w)	WS (m_c)	PI (p_i)	V (v)	RE (R_e)	EU (E_u)	FW (f_w)	I-PHI ($1-\Phi$)
0	1955.	0.0062	0.1158E05	0.2049E01	0.4159E05	0.	0.1567E-00	0.7307E00
0.03540	0.	0.	0.1108E05	0.3578E01	0.1131E06	0.1967E-01	0.2185E-01	0.3315E-01
0.08660	0.	0.	0.1056E05	0.6300E01	0.3852E06	0.8113E-01	0.3807E01	0.1956E-01

TURBULENT FLOW REGION   X = 0.15500   D = 0.00850

Y	TW	WS	PI	V	RE	EU	FW	I-PHI
0.	1882.	0.00100	0.1226E05	0.3175E01	0.4398E05	0.	0.2497E-00	0.6336E00
0.03540	0.	0.	0.1174E05	0.4547E01	0.1131E06	0.1967E-01	0.277E-01	0.1670E-01
0.08660	0.	0.	0.1121E05	0.7244E01	0.3852E06	0.8113E-01	0.4377E-01	0.1343E-01

NOTE: A number such as 0.4159E-05 means 0.4159 x 10^5

TABULATION OF COMPUTED RESULTS

TABLE III

References

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